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Influence of the r-mode instability on hypercritically accreting neutron stars

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ABSTRACT

We have investigated an influence of the r-mode instability on hypercritically accreting ($\dot{M} \sim 10^8 \dot{M}_{\rm Edd}$) neutron stars in close binary systems during their common envelope phases based on the scenario proposed by Bethe et al. (1999). On the one hand neutron stars are heated by the accreted matter at the stellar surface, but on the other hand they are also cooled down by the neutrino radiation. At the same time, the accreted matter transports its angular momentum and mass to the star. We have studied the evolution of the stellar mass, temperature and rotational frequency.

The gravitational-wave-driven instability of the r-mode oscillation strongly suppresses spinning-up of the star, whose final rotational frequency is well below the mass-shedding limit, typically as small as 10% of that of the mass-shedding state. On a very short timescale the rotational frequency tends to approach a certain constant value and saturates there as far as the amount of the accreted mass does not exceed a certain limit to collapse to a black hole. This implies that the similar mechanism of gravitational radiation as the so-called Wagoner star may work in this process. The star is spun up by accretion until the angular momentum loss by gravitational radiation balances the accretion torque. However the lifetime of the system is extremely short, roughly one year or so, to be detected by gravitational wave detectors.

Key words: accretion – binaries: close – stars: neutron – stars: rotation – stars: oscillations

1 INTRODUCTION

In relation to the formation mechanism of compact binary systems such as double neutron star systems, double black hole systems or black hole – neutron star systems, hypercritical accretion flows onto neutron stars in envelopes of massive stars of close binary systems have been investigated by several authors (Chevalier 1993; Chevalier 1996; Brown 1995; Bethe & Brown 1998; Bethe et al. 1999). The mass accretion rate ($\dot{M}$) of the hypercritical accretion flow amounts to 10^8 times as high as the Eddington's mass accretion rate ($\dot{M}_{\rm Edd}$). Chevalier (1996) found that for rather small values of the viscosity parameter, $\alpha \sim 10^{-6}$, the accretion flow which cools via neutrino radiation could be realized.

Recently Bethe et al. (1999) pointed out that the flow with much larger and conservative values of the viscosity parameter, $\alpha \sim 0.1$, does not become so hot as to emit neutrinos during the accretion process but that the accretion will proceed to the advection dominated accretion flow (ADAF). In their model, the accreted matter onto the neutron star surface is processed there by nuclear reactions and cools via neutrino emission. The hypercritical accretion increases the mass of the star rapidly and induces the collapse of the neutron star to a black hole. Thus the supernova explosion of the massive companion will lead to the formation of a black hole - neutron star binary system or a double black hole binary system. The merging event of such a system might be one of the important targets for the gravitational wave astronomy.

In the accretion process, a non-negligible amount of angular momentum is carried into the neutron star. This potentially could spin the star up to the Kepler limit, i.e. to the state where the stellar rotational frequency equals to the orbital frequency at the equatorial surface. The star rotating at the Kepler limit deforms significantly from the spherical configuration and the subsequent collapse of the rapidly rotating star to a black hole might radiate a substantial fraction of the binding energy of the system in the form of gravitational waves.

This seemingly straightforward conclusion, however, has to be reexamined from the standpoint of stability of rotating neutron stars. The gravitational radiation driven

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instability of rotating stars might prevent the stellar rotational frequency to increase above a certain limit. In particular, the recently discovered r-mode instability (Andersson 1998; Friedman & Morsink 1998) might be sufficiently strong to suppress the spin frequency in some range of the stellar temperature.

In this paper, in order to see how this instability influences the accretion induced spin-up of the neutron star in the hypercritical accretion, we will employ the hypercritical ADAF model based on the analysis of Bethe et al. (1999) and investigate the evolution of the temperature and the rotational frequency of the neutron star. †

2 SIMPLIFIED MODEL OF THE ACCRETION INDUCED EVOLUTION OF NEUTRON STARS

First it should be noted that the study in this paper is done in the framework of Newtonian dynamics and Newtonian gravity. The gravitational radiation reaction on stellar oscillations is evaluated by the lowest order post-Newtonian approximation (Thorne 1980). Furthermore to simplify our analysis, stellar configurations are assumed to be spherical polytropes with the polytropic index $N = 1$.

2.1 Characteristics of the hypercritical ADAF onto neutron stars

The hypercritical accretion flow which we employ in this paper is proposed by Bethe et al. (1999). The characteristics of that flow are summarized as follows. (1) The flow is hypercritical because the accretion rate is typically $\sim 1 M_\odot \mathrm{y}^{-1}$. (2) Since the flow is advection dominated (Narayan & Yi 1994) and its temperature is relatively low (~ 0.5MeV), the adiabatic exponent of the gas equals to 4/3 (radiation dominated mixture of photon, electron, positron and nuclei). (3) The accreted matter touches down on the surface of the star 'softly' and burns to produce as heavy nuclei as ^{56}Ni. On the other hand, the matter is cooled by neutrino emission. Consequently the whole process acts as a 'thermostat' which keeps the neutrino temperature ~ 1MeV.

2.2 Mass accretion rate

The mass accretion rate $\dot{M}_{\mathrm{acc}}$, which is the rate of mass attachment to the star, is assumed to be constant during the evolution. In general the mass accretion rate should be distinguished from the mass inflow rate $\dot{M}_{\mathrm{in}}$, which is the total amount of the incoming mass per unit time to the neutron star. This is because some kinds of outflow may coexist with the accreting flows. In this paper we simplify the situation by assuming that both rates are the same, i.e. $\dot{M}_{\mathrm{in}} = \dot{M}_{\mathrm{acc}} \equiv \dot{M}$. We will discuss this assumption further in *Discussion*.

The evolution of the neutron star mass M is written as:

$$\frac{M}{M_\odot} = \mu_0 + \beta t \ , \tag{1}$$

where μ_0 is the initial mass of the star (in units of the solar mass), time t is measured in units of second, and β is defined as:

$$\beta = 3.2 \times 10^{-8} \left[\frac{\dot{M}}{1 M_\odot \mathrm{y}^{-1}} \right] \ . \tag{2}$$

2.3 Thermal response of the neutron star heated by neutrino from the surface

The accreted matter soft-landing on the surface layer is processed by nuclear burning and emits neutrinos. We will assume that roughly a half of them escapes from the star freely and that some fraction of the rest of them interacts with stellar matter and heats it up. The mean free path λ of inelastic scattering of neutrinos with electrons in the neutron star matter is written as (Shapiro & Teukolsky 1983),

$$\lambda \sim 2 \times 10^{10} \left(\frac{\rho_{\mathrm{nuc}}}{\rho} \right)^{7/6} \left(\frac{0.1\mathrm{MeV}}{E_\nu} \right)^{5/2} \mathrm{cm} \ , \tag{3}$$

where ρ_{nuc} is the 'nuclear density' ($\sim 2.8 \times 10^{14}$g cm^{-3}) and E_ν is the neutrino energy. The density ρ means the averaged density of the star, which is related in our polytropic stellar models to the mass M as follows:

$$\frac{\rho}{\rho_{\mathrm{nuc}}} = 0.7 \frac{M}{M_\odot} \ . \tag{4}$$

The neutrino energy E_ν is fixed to 1MeV. We simply assume the scattering (thus heating) rate as $\eta R/\lambda$ where R is the stellar radius and η is an efficiency factor of ν-heating through the inelastic scattering. ‡ As the flow is adiabatic and radiation dominated, the specific internal energy of the accreted matter ε is written as,

$$\varepsilon = \frac{3}{4} \times \frac{6}{7} \frac{GM}{R} \ . \tag{5}$$

With these expressions, the heating rate of the whole star $\dot{\varepsilon_{\mathrm{h}}}$ is,

$$\begin{aligned} \dot{\varepsilon_{\mathrm{h}}} &\sim \dot{M} \varepsilon \frac{\eta R}{\lambda} \\ &\sim 4.7 \times 10^{43} \left(\frac{\dot{M}}{1 M_\odot \mathrm{y}^{-1}} \right) \left(\frac{M}{M_\odot} \right)^{13/6} \mathrm{erg\ s}^{-1}. \end{aligned} \tag{6}$$

Here we choose $\eta = 1$. Heating rate above is proportional to the multiplication of $\dot{M}$ and η, thus the reduction of η is equivalent to that of $\dot{M}$ with η being fixed. As seen in later discussion, the dependence of the results on η is weak. As the cooling mechanism of the star, we assume the modified URCA process to be dominant (Shapiro & Teukolsky 1983). Its cooling rate L_ν^{URCA} is,

$$L_\nu^{\mathrm{URCA}} = 6 \times 10^{39} \left(\frac{M}{M_\odot} \right)^{2/3} T_9^8 \ , \tag{7}$$

where T_9 is the stellar temperature, which is assumed to be uniform, in units of 10^9K. The heat capacity C_{v} of the

† Similar investigations of accreting neutron stars with much lower accretion rate, which are the models of LMXBs, can be seen for instance in Andersson et al. (1999) and in Levin (1999).

‡ Note that in our $N = 1$ polytropic model, R is constant and this is fixed to 12.5km.

neutron star which is approximated to consist of degenerate free nucleons is,

$$C_{\rm v} = 8 \times 10^{54} k_B \times \left(\frac{M}{M_\odot}\right) \left(\frac{\rho}{\rho_{\rm nuc}}\right)^{-2/3} T_9 \ , \qquad (8)$$

where k_B is the Boltzmann constant. Then the equation of the thermal balance of the star can be written as,

$$\frac{d}{dt}\left(\frac{1}{2} C_{\rm v} T\right) = \dot{\varepsilon_{\rm h}} - L_\nu^{\rm URCA} \ . \qquad (9)$$

This is transformed to the equation of the evolution of the temperature as follows:

$$\frac{d}{dt}\left[\left(\frac{M}{M_\odot}\right)^{1/3} T_9^2\right] = 6.5 \times 10^{-5} \left[\frac{\dot{M}}{1 M_\odot {\rm y}^{-1}}\right] \left(\frac{M}{M_\odot}\right)^{13/6} - 8.7 \times 10^{-9} \left(\frac{M}{M_\odot}\right) T_9^8 \ . \qquad (10)$$

2.4 Angular momentum balance of the star

The angular momentum of the star is changed by two torques induced by accretion and by gravitational radiation if the r-mode oscillation is unstable.

The accretion torque is estimated as $\dot{M} j_a$, where j_a is the specific angular momentum of the accreted matter:

$$j_a = R \sqrt{\frac{2}{7} \frac{GM}{R}} \ . \qquad (11)$$

Here the ADAF solution by Narayan & Yi (1994) is employed to evaluate the rotational velocity of the accreted matter at the stellar surface.

As for the gravitational radiation torque, we use the simple formula used in Lindblom et al. (1998) which also incorporates the damping effect by viscosity. This corresponds to the assumption that the r-mode quickly grows to saturate in the non-linear regime of its amplitude and the angular momentum of the mode is a substantial fraction of the stellar angular momentum. Using the stellar moment of inertia I and the stellar rotational angular frequency Ω, this torque can be calculated as $\tau_{\rm r}^{-1} I \Omega$. The timescale of the r-mode instability $\tau_{\rm r}$ is defined as,

$$\begin{aligned} \tau_{\rm r}^{-1} &= \tau_{\rm gr}^{-1} + \tau_{\rm s}^{-1} + \tau_{\rm b}^{-1} \\ &= -6.4 \times 10^{-9} \left(\frac{f}{100{\rm Hz}}\right)^6 \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10{\rm km}}\right)^7 \\ &+ 3.2 \times 10^{-9} \left(\frac{M}{M_\odot}\right)^{5/4} \left(\frac{R}{10{\rm km}}\right)^{-2} T_9^{-2} \\ &+ 1.4 \times 10^{-11} \left(\frac{f}{100{\rm Hz}}\right)^2 \left(\frac{M}{M_\odot}\right)^{-1} T_9^6 \ , \qquad (12) \end{aligned}$$

where $\tau_{\rm gr}$ is the energy dissipation time scale by the gravitational radiation, $\tau_{\rm s}$ is that of the shear viscosity due to neutron-neutron collision and $\tau_{\rm b}$ is that of the bulk viscosity due to the lag of the $\beta-$reaction and the stellar oscillation. To evaluate the gravitational radiation timescale, the $l = m = 2$ r-mode which contributes mainly to the instability is solely taken into account. Here f is the rotational frequency of the star ($f = \Omega/2\pi$). The negative value of $\tau_{\rm r}$ signals the onset of the instability. We set $\tau_{\rm r}^{-1}$ to be zero when this value is positive, i.e. for damping oscillations.

The equation for the evolution of the rotational frequency is written as,

$$\frac{d}{dt}\left[\left(\frac{M}{M_\odot}\right)\left(\frac{f}{100{\rm Hz}}\right)\right] = \tau_r^{-1} \left(\frac{M}{M_\odot}\right)\left(\frac{f}{100{\rm Hz}}\right) + 3 \times 10^{-6} \left[\frac{\dot{M}}{1 M_\odot {\rm y}^{-1}}\right] \left[\frac{R}{10{\rm km}}\right]^{-9/5} \left(\frac{M}{M_\odot}\right)^{1/2} . \qquad (13)$$

2.5 Terminal values of the temperature and the rotational frequency of nearly constant mass models

First it is important to note that the thermal balance as well as the angular momentum balance determines the approximate terminal values of the temperature and the rotational frequency of the star, if the accretion rate is fixed and the mass is regarded as nearly constant in the evolution.

By setting the left hand side of Eq. (10) to be zero, the temperature can be solved as,

$$T_9 = 3.0 \left[\frac{\dot{M}}{1 M_\odot {\rm y}^{-1}}\right]^{1/8} \left(\frac{M}{M_\odot}\right)^{7/48} . \qquad (14)$$

Thus for the star with mass $M \sim 1.4 M_\odot$ and the accretion rate with $\dot{M} \sim 1 M_\odot {\rm y}^{-1}$, the (uniform) temperature will be about 3.5×10^9K, which is near the most susceptible temperature for the r-mode instability to grow (Lindblom et al. 1998).

The equation of the angular momentum balance gives the terminal value of the rotational frequency of the star by setting the right hand side of Eq.(13) to be zero. If we omit the contributions of viscous damping whose effect is smallest near at the temperature obtained above, we get the expression for the terminal rotational frequency as follows:

$$\frac{f}{100{\rm Hz}} = 1.7 \left[\frac{\dot{M}}{1 M_\odot {\rm y}^{-1}}\right]^{1/7} \left[\frac{R}{10{\rm km}}\right]^{-44/35} \left(\frac{M}{M_\odot}\right)^{-3/14} . \qquad (15)$$

2.6 Evolutionary curves

Given the mass accretion rate and the initial values of the stellar mass, temperature and frequency, the evolution of the corresponding quantities can be followed by integrating Eqs. (1), (10) and (13). We have computed several evolutionary sequences by choosing different sets for the initial values.

First we will show how the evolution depends on the initial values. In Fig. 1 the rotational frequency is plotted against the mass of the star which serves as the 'time' variable because the mass accretion rate is assumed to be constant. With the mass accretion rate fixed, solutions with different values of the initial frequency converges to a certain value more quickly compared with the mass accretion timescale. This behaviour is expected from the rapid temperature saturation seen in Fig. 2 by swift establishment of the thermal balance between heating and cooling. It should be pointed out that the omission of the r-mode instability would lead to very rapid increase of the rotational frequency and that the star would reach its mass-shedding limit before the accretion induced collapse occurs.

The mass accretion rate dependence of the frequency

Figure 1. Evolutionary curves of the rotational frequency $f/100\mathrm{Hz}$ for different values of the initial rotational frequency, i.e. $f/100\mathrm{Hz} = 0.1, 0.5, 1$ and 3, are plotted against the stellar mass which can be regarded as the 'time' variable. Model parameters are as follows: the initial mass $M = 1.4M_\odot$, the initial temperature $T_9 = 10^{-3}$, the mass accretion rate $\dot{M} = 1M_\odot \mathrm{y}^{-1}$. Numbers attached to the curves are the values of the lapse of time in units of second. Also plotted is the curve of the model in which the r-mode instability is not included (dashed curve).

Figure 2. Evolutionary curves of the temperature $T_9 = T/10^9(\mathrm{K})$ for different values of the mass accretion rate, i.e. $\dot{M}/1M_\odot \mathrm{y}^{-1} = 1$ (solid curve), 10^{-1} (dotted curve), 10^{-2} (dashed curve), are plotted against the stellar mass. Model parameters are as follows: the initial mass $M = 1.4M_\odot$, the initial temperature $T_9 = 10^{-3}$, and the initial rotational frequency $f/100\mathrm{Hz} = 0.5$.

Figure 3. The rotational frequency is plotted against the stellar mass. Three lines correspond to the three temperature lines in Fig. 2 ($\dot{M}/1M_\odot \mathrm{y}^{-1} = 1$ (solid curve), 10^{-1} (dotted curve), 10^{-2} (dashed curve)). Numbers attached to these lines are the values of the lapse of time in units of second.

evolution is shown in Fig. 3. As seen in Fig. 2 the temperature of the star quickly saturates at the limiting value which depends on the mass accretion rate. Since the strength of the r-mode instability depends on the temperature as well as the stellar rotational frequency, the rotational frequency also quickly saturates. As the timescale of the angular momentum change is longer than that of the temperature change, the frequency saturation is reached slightly later compared with the temperature saturation.

3 DISCUSSIONS

We have shown that the r-mode instability sets a severe limit on the rotational frequency of the hypercritically accreting neutron stars proposed by Bethe et al. (1999). As the mass accretion rate dependence of the rotational frequency (Eq. (15)) is rather weak, the rotational frequency of the star is well below its mass-shedding limit. Even if the mass accretion rate rises to $\dot{M} = 10^2 M_\odot \mathrm{y}^{-1}$, the rotational frequency increases only to 200Hz. Thus the gravitational collapse of neutron stars due to mass accretion to low mass black holes in these systems will proceed almost spherically and black holes formed just after the collapse will rotate so slowly that the rotational parameter is not large either, i.e. $q = cJ/GM^2 \sim 0.01$ for $1.7M_\odot$.

For simplicity in this study we apply 'one zone' approximation in which each variable characterizing a stellar model can have one value. This means that the variables such as stellar temperature or rotational frequency is represented by some 'averaged' values. Compared with the case of low accretion rate binary, the evolution of the system proceeds much faster, which typically takes one year. The timescale of shear viscosity to smooth out the differential flow can be estimated as (for the neutron-neutron scattering dominant matter (Lindblom et al. 1998)),

$$\tau \sim \frac{R^2 \bar{\rho}}{\eta} \sim \frac{R^2 T^2}{347 \bar{\rho}^{5/4}}. \tag{16}$$

Assuming the stellar radius $R = 10$km and the average density $\bar{\rho} = 3 \times 10^{14}$g/cm^3, this amounts to 0.8y for $T = 10^8$K. Thus the rotation law of the star can be substantially apart from the uniform rotation. In that case, behaviour of (classical) r-modes are uncertain. Some kind of mode mixing may occur and the characteristics of them may be drastically changed. We here *assume* that they can be approximated by some kind of averaging on the rotational angular velocity. The validity of this assumption should be confirmed elsewhere.

It has been pointed out that in the hypercritical accretion process, explosive outflows driven by neutrino heating (Fryer et al. 1998) or jet formation (Armitage & Livio 1999) may significantly reduce the mass attachment onto the neutron star. If it is the case, it is more probable that double neutron star binary systems will be formed through this channel. In such accretion inefficient situations, the mass accretion rate $\dot{M}$ is lower than the mass inflow rate $\dot{M}_{\rm in}$ from the envelope of the companion star. If the accretion inefficient flows behave similarly as the accretion efficient flows, evolution of the stars may be approximated by our models with lower $\dot{M}$ and with a shorter duration, for the same value of $\dot{M}_{\rm in}$. This results in smaller terminal rotational frequencies of the star. Thus neutron stars which survive after mass accretion in this process would have even lower rotational frequencies. [§]

Finally we will evaluate the amount of gravitational radiation from the system as follows. The process discussed in this paper is similar to that studied by Wagoner (1984). A neutron star in an X-ray binary system emits gravitational wave so as to balance the accretion torque with the gravitational radiation torque of f-mode ('Wagoner star'). For our models the f-mode must be replaced by the r-mode and the accretion rate is considerably different, i.e. increased. The standard formula of the luminosity of gravitational radiation can be expressed as (Shapiro & Teukolsky 1983),

$$\frac{c^3}{G} \omega^2 h^2 D^2 \sim 2\pi f \dot{M} j_a \ , \tag{17}$$

where ω, h, D are the eigenfrequency of the mode, the dimensionless strain of the gravitational field and the distance from the source, respectively. From this equation we get

$$h \sim 10^{-23} \left[\frac{\dot{M}}{1 M_\odot {\rm y}^{-1}} \right]^{3/7} \left(\frac{M}{M_\odot} \right)^{5/14} \left(\frac{D}{10{\rm kpc}} \right)^{-1}. \tag{18}$$

However, since the lifetime of the system, typically ~ 1y, is much shorter than the ordinary Wagoner stars, the event rate expected, $\sim 10^{-4}$y^{-1}galaxy^{-1} (Bethe & Brown 1998), is too small to be detected.

§ It is possible that the characteristics of the accretion flow change drastically and that the accreted matter carries less angular momentum than the case considered here. Of course this will lead to the even more reduction of the terminal rotational frequency.

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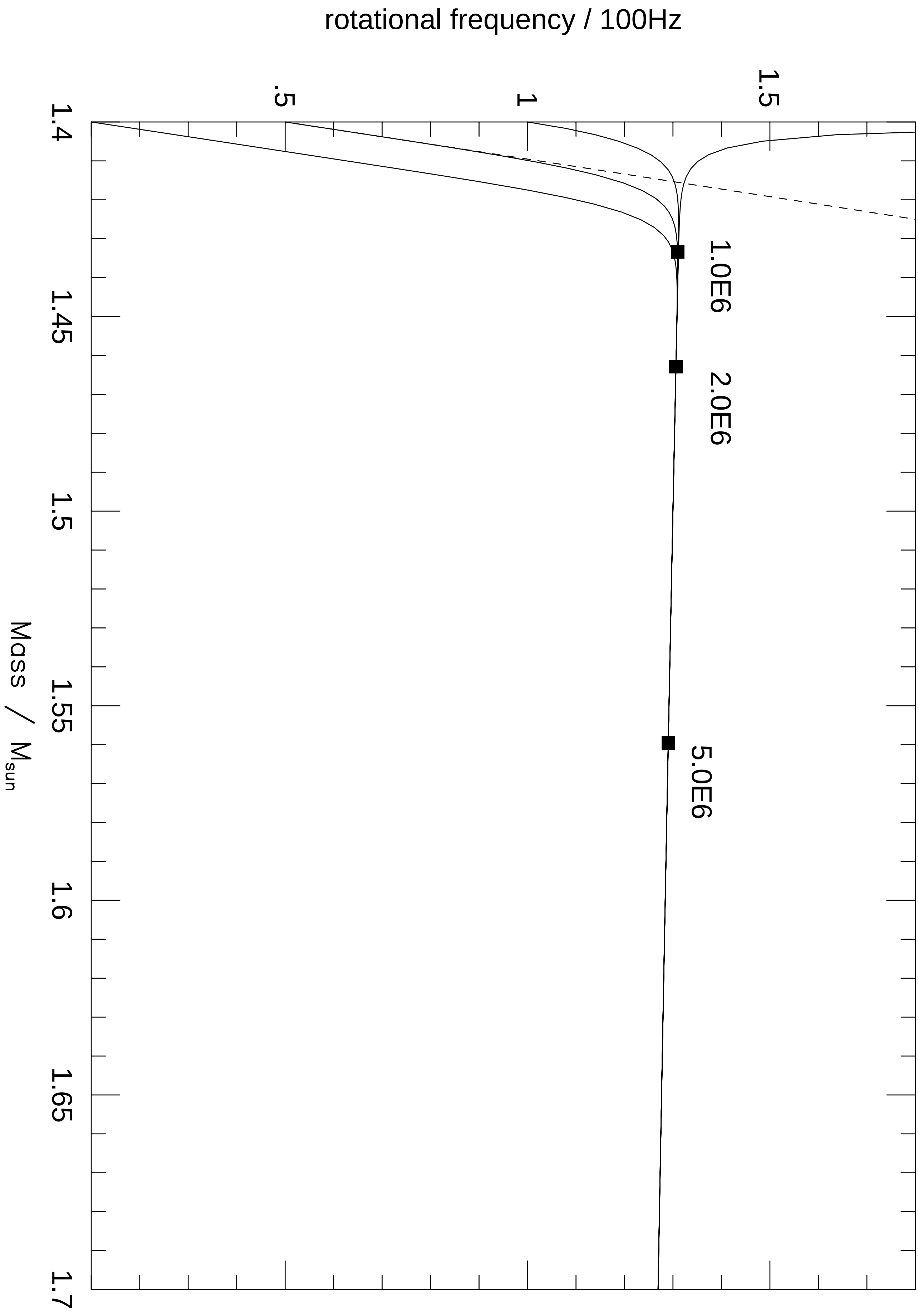
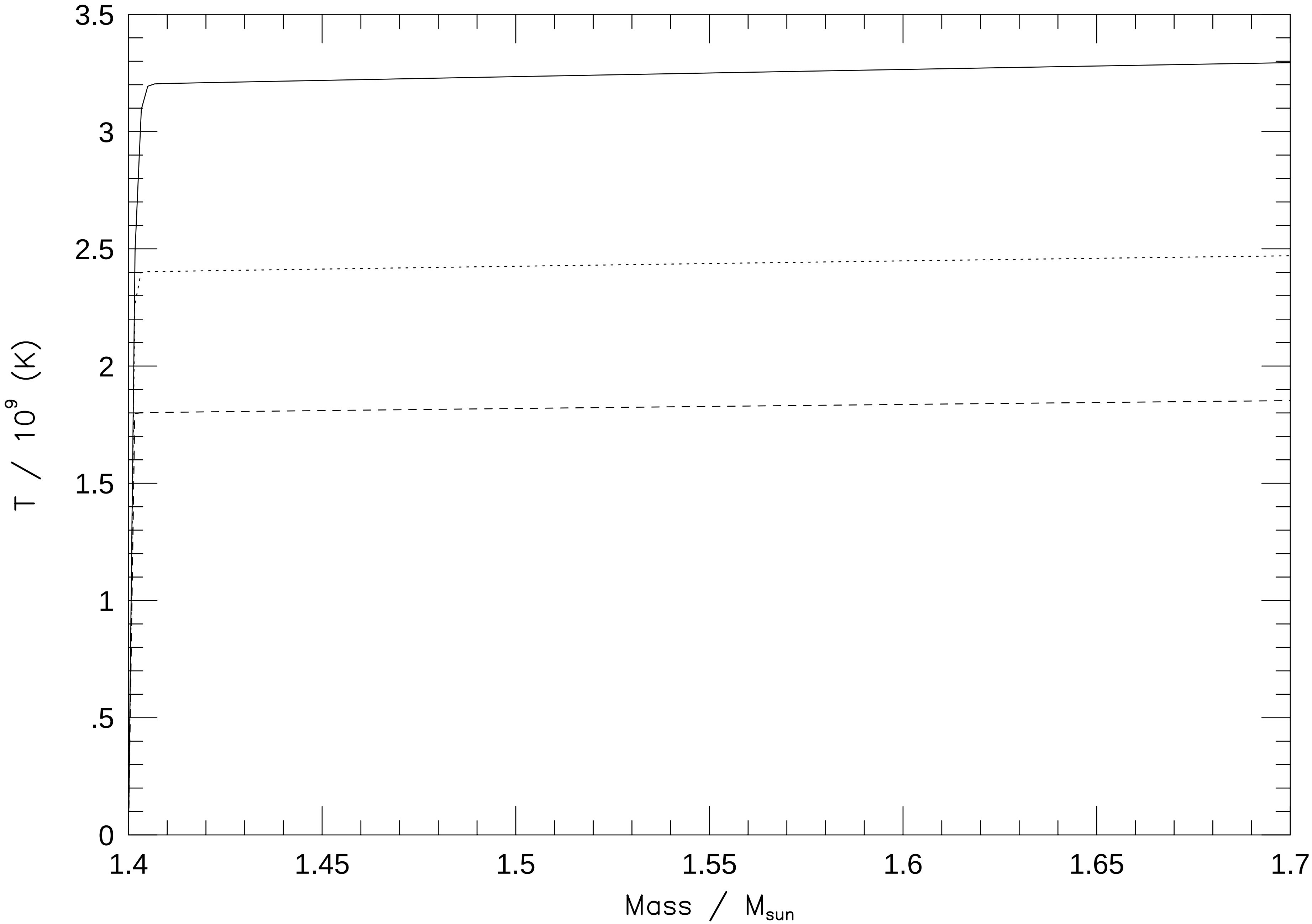


rotational frequency / 100Hz
.5
1
1.5
1.4
1.45
1.5
1.55
1.6
1.65
1.7
Mass / M_{sun}
1.0E6
2.0E6
5.0E6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2.5
2
1.5
1
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T / 10^9 (K)
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1.5
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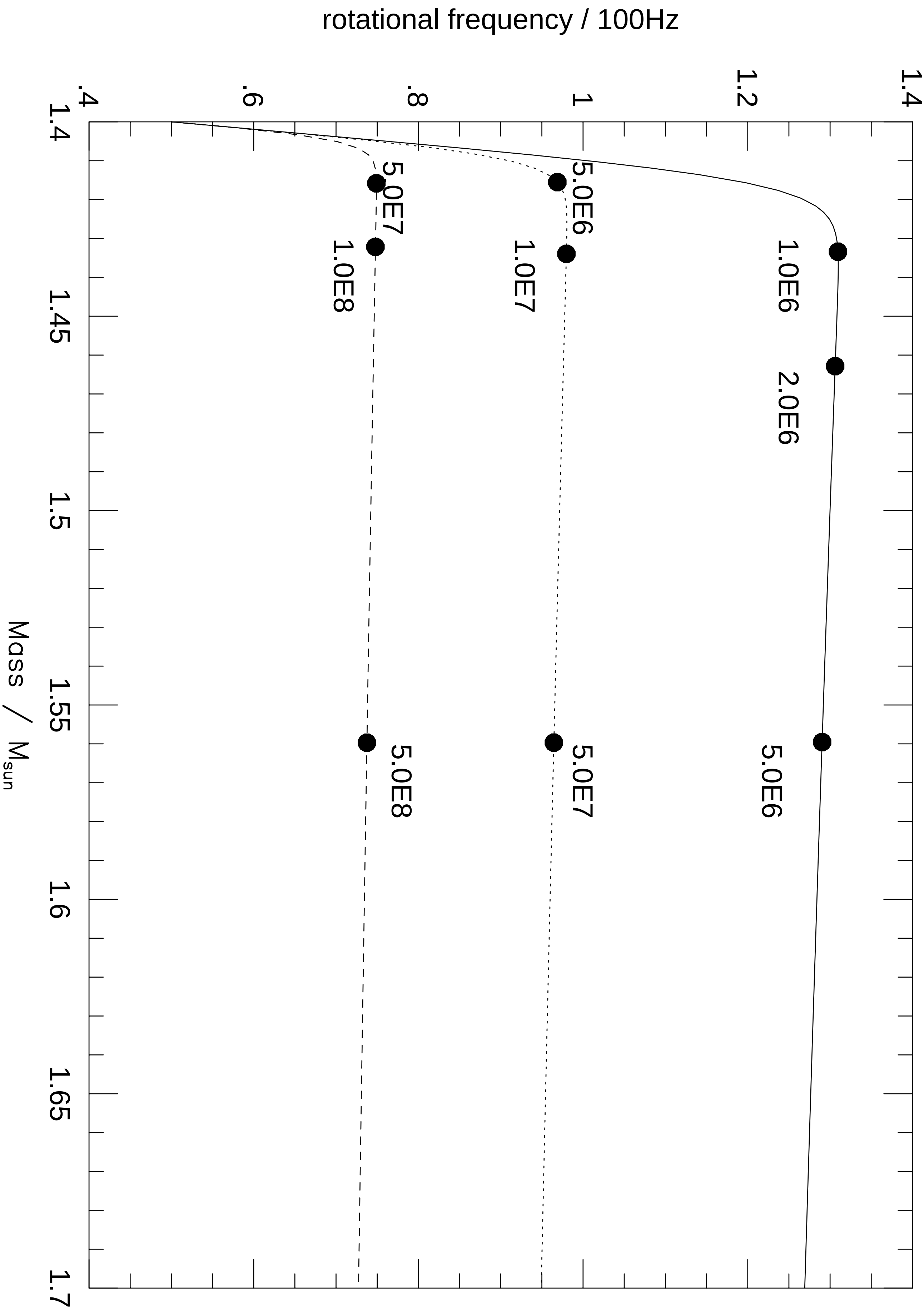
1.65
1.7
Mass / Msun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1.4
1.4
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1.5
1.55
1.6
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1.7
Mass / M_{sun}
5.0E7
1.0E8
5.0E8
5.0E6
1.0E7
5.0E7
1.0E6
2.0E6
5.0E6